CONTROL OF CHAOTIC SYSTEMS

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Summary

The field related to control of chaotic systems was rapidly developing during the 1990s. Its state-of-the-art in the beginning of the 21st century is presented in this article. Necessary preliminary material is given related to notion and properties of chaotic systems, models of the controlled plants and control goals. Several major branches of research are discussed in detail: feedforward or “nonfeedback” control (based on periodic excitation of the system); OGY method (based on linearization of Poincaré map); Pyragas method (based on time-delay feedback); traditional control engineering methods of linear, nonlinear and adaptive control; neural networks; fuzzy control. Some unsolved problems concerning the justification of chaos control methods are presented. Other directions of research are outlined such as control of distributed (spatio-temporal and delayed) systems, chaotic mixing, generation of chaos (chaotization), etc. Areas of

existing and potential applications in science and engineering are pointed out.

1. Introduction

Chaotic system is a deterministic dynamical system exhibiting irregular, seemingly random behavior. Two trajectories of a chaotic system starting close to each other will diverge after some time (so-called “sensitive dependence on initial conditions”). Mathematically chaotic systems are characterized by local instability and global boundedness of the trajectories. Since local instability of a linear system implies unboundedness (infinite growth) of its solutions, chaotic system should be necessarily nonlinear, i.e. should be described by a nonlinear mathematical model.

Control of chaos, or control of chaotic systems, is the boundary field between control theory and dynamical systems theory studying when and how it is possible to control systems exhibiting irregular, chaotic behavior. Control of chaos is closely related to nonlinear control, and many methods of nonlinear control are applicable to chaotic systems. However control of chaotic systems has some specific features.

A key property of chaotic systems is its instability: sensitive dependence on initial conditions. An important consequence is high sensitivity with respect to changes of input (controlling action). It means that small changes of control may produce large variations in systems behavior. Such a phenomenon and its implications in physics were described in the seminal paper of 1990 by E. Ott, C. Grebogi, J. Yorke from the University of Maryland, USA that triggered an explosion of activities and thousands of publications during the following decade.

A typical control goal when controlling chaotic systems is to transform a chaotic trajectory into a periodic one. In terms of control theory it means stabilization of an unstable periodic orbit or equilibrium. A specific feature of this problem is the possibility of achieving the goal by means of an arbitrarily small control action. Other control goals like synchronization and chaotization can also be achieved by small control in many cases.

For almost three decades after the term “chaos” was coined, chaotic phenomena and chaotic behavior have been observed in numerous natural and model systems in physics, chemistry, biology, ecology, etc. Paradigm of chaos allows us to better understand inherent properties of natural systems. Engineering applications are rapidly developing in areas such as lasers and plasma technologies, mechanical and chemical engineering and telecommunications. Possibilities of controlling complex behavior by means of small control open new horizons both in science and in technology.

Development of new methods for control of chaos or “control by tiny corrections” may be of utmost importance for sustained development of humanity. They may be efficient for solving problems where applying stronger control is not possible either because of lack of resources (like in many large scale systems: economies, energy systems, weather control, etc.) or because intervening the natural dynamics is undesirable (e.g. in biological and biomedical applications, ecological systems).

It is worth noticing that, in spite of the enormous number of published papers, not many rigorous mathematical results are so far available. A great deal of results is justified by computer simulations rather than by analytical tools and many problems remain unsolved. Main approaches to controlling chaotic behavior are described below. Before exposition of the methods some preliminaries are given concerning system models, control goals and properties of chaotic systems.

2. Notion of Chaos

There exist different formal definitions of a chaotic system underlying different features of chaotic behavior. Loosely speaking, chaotic processes are defined as solutions of nonlinear differential or difference equations, characterized by local instability and global boundedness. Their main feature is that the solutions with arbitrarily close initial conditions diverge to a rather large distance after some time (so-called "sensitive dependence on initial conditions"). Formal definitions are based on appropriate formalizations of stability concept. Below a typical definition and a typical criterion of chaos are introduced. More details can be found in *Modeling and analysis of chaotic systems*.

Consider a dynamical system described by the differential equation

$$\dot{x} = F(x), \tag{1}$$

where $x \in \Re^n$ is n-dimensional state vector, $\dot{x} = d/dt$ stands for the time derivative of x. Let $\overline{x}(t)$, $0 \le t < \infty$ be a solution of the system (1) with initial condition $\overline{x}(0) = \overline{x}_0$. To define a chaotic system the notions of *attracting set*, *attractor* and a *chaotic attractor* are used.

A set B is called an *attracting set* for system (7) if there exists an open set $B_0 \supset B$ such that all the solutions $x(t)$ starting from the set B_0 exist for all $t \ge 0$ and $\lim_{t\to\infty} dist\left(x(t), B\right) = 0$ for any solution $x(t)$ with $x(0) \in B_0$. The set of initial conditions B_0 for which (8) holds is called *basin of attraction*. A closed attracting set B is called *attractor* if it is minimal, i.e. there is no smaller attracting subset of B. An attractor B is called *chaotic* if it is bounded and all the trajectories starting from it are Lyapunov unstable. Finally, a system (1) is called *chaotic* if it possesses at least one chaotic attractor. Similar definitions are introduced for discrete-time system $x_{k+1} = f(x_k)$, $k = 0,1,\ldots$

Because of coexistence of different definitions of chaos, theoretical studies are often based only on some features of chaotic systems, without specifying a rigorous definition. An important feature of chaotic trajectories for many purposes is *recurrence*: they return to any vicinity of any past value of the trajectories.

Since formal verification of chaoticity of a system behavior is usually very difficult, various numerical criteria are used. The most common criterion of chaotic behavior for a system is the positivity of its largest Lyapunov exponent. For a linear system $\dot{x} = A(t)x$ the largest Lyapunov exponent ρ_{L} is defined as follows:

$$\rho_{\mathrm{L}} = \overline{\lim_{t\to\infty}} \frac{\ln\left|\mathbf{\Phi}(t,t_0)\right|}{t-t_0}, \qquad (2)$$

where $\mathbf{\Phi}(t,\tau)$ is the fundamental matrix of system $\dot{\mathbf{x}} = \mathbf{A}(t)\mathbf{x}$ satisfying $x(t) = \Phi(t,\tau)x(\tau)$ for all $t,\tau \in \Re^1$, $\Phi(0,0) = I$.

Hereafter $|x|$ stands for Euclidean norm of a vector or a matrix $|x|$, I denotes the unit matrix of appropriate size.

For a nonlinear system the largest Lyapunov exponent depends on a prespecified (base) solution $\bar{x}(t)$ and for given $\bar{x}(t)$ it can be defined as the largest Lyapunov exponent of the system linearized near $\bar{x}(t)$ (i.e. the linear system $\dot{x} = A(t)x$ with the matrix $A(t) = \partial F\left(\bar{x}(t)\right)/\partial t$). (See *Modeling and analysis of chaotic systems*)

3. Models of Controlled Systems and Control Goals

Models of controlled systems. A formal statement of a control problem typically begins with a model of the system to be controlled (*controlled system* or *controlled plant*) and a model of the control objective (*control goal*). If the plant model is not given *a priori* (as in many real life applications) some approximate model should be determined in some way. Several classes of models are considered in the literature related to control of chaos. The most common class consists of continuous systems with lumped parameters described in state space by differential equations

$$\dot{x} = F(x,u), \qquad (3)$$

where x is n-dimensional vector of the state variables; u is m-dimensional vector of inputs (control variables). The vector–function $F(x,u)$ is usually assumed continuously differentiable which guarantees local existence and uniqueness of solutions of (3). The model should also include the description of measurements, i.e. the l-dimensional vector of output variables y should be defined, for example

$$y = h(x). \qquad (4)$$

If the outputs are not defined explicitly, it is assumed that all the state variables are available for measurement, i.e. $y = x$.

The model (3) encompasses two physically different cases:

A. The input variables represent some physical variables (forces, torques, intensity of electrical or magnetic fields, etc.). For example a model of a controlled oscillator (pendulum) can be put into the form

$$J\ddot{\varphi}+r\dot{\varphi}=ml\sin\varphi=u(t), \tag{5}$$

where φ is the angle of deflection from vertical; J,m,l are physical parameters of the pendulum (inertia, mass, length); $u(t)$ is a controlling torque. The description (5) is transformable into the form (3) with the state vector $x=(\varphi,\dot{\varphi})^{\mathrm{T}}$.

B. The input variables represent change of physical parameters of the system, i.e. $u(t)=p-p_0$, where p_0 is the nominal value of the physical parameter p. For example, in the case when the pendulum is controlled by changing its length, its model, instead of (5), becomes

$$J\ddot{\varphi}+r\dot{\varphi}=m\big(l_0+u(t)\big)\sin\varphi=0, \tag{6}$$

where l_0 is the initial length of the pendulum.

Although from a physical point of view the difference between the cases A and B does exist, for the purpose of studying the nonlinear system (3) this difference can be neglected.

If external disturbances are present, more general time-varying models

$$\dot{x}=F(x,u,t) \tag{7}$$

are considered. Often more simple affine in control models

$$\dot{x}=f(x)+g(x)u \tag{8}$$

can be employed.

It is convenient for the purposes of analysis to represent a nonlinear system in the so-called *Lurie form*: system consisting of a linear part

$$\dot{x}=Ax+Bu, \quad y=Cx \tag{9}$$

where A,B,C are constant matrices of appropriate dimensions with a static nonlinearity

$$u=\varphi(y). \tag{10}$$

If the nonlinear part satisfies some input-output relations, e.g. sector constraints:

$$|\varphi(y)| \le K_\varphi |y|, \tag{11}$$

then for analysis of Lurie systems efficient *frequency-domain methods* based on examination of the frequency response $\mathrm{Re}W(j\omega)$, where $W(\lambda) = C(\lambda I - A)^{-1} B$ is the transfer function of the linear part (9), $j = \sqrt{-1}$ can be employed.

Many real world systems can be described by discrete-time state-space models

$$x_{k+1} = F_d(x_k, u_k). \tag{12}$$

where $x_k \in \Re^n, u_k \in \Re^m, y_k \in \Re^l$, are the values of the state, input and output vectors at the kth stage of the process, respectively. Since the model (12) can be described by specifying the map F_d, in the literature the term “control of the map” is often used instead of “control of the discrete-time system”.

In some cases delay-differential models

$$\dot{x} = F(x(t), x(t-\tau), u(t), u(t-\tau_u)), \tag{13}$$

and delay-difference models

$$x_{k+1} = F_\mathrm{d}(x_k, x_{k-1}, \ldots, x_{k-\tau}, u_k, \ldots, u_{k-\tau_u}) \tag{14}$$

are used. To determine solutions of system (13) on some time interval $[t_0, t_1]$ it is necessary to specify the initial state function $\bar{X}_0 = \{x(s), t_0 - \tau \le s \le t_0\}$ in addition to the input function $\bar{U}_0 = \{u(s), t_0 - \tau \le s \le t_0\}$. In what follows it is assumed that all the models under consideration satisfy conditions guaranteeing existence of the their solutions starting from given initial conditions for all $t \ge t_0$. For simplicity we will also assume that $t_0 = 0$ whenever possible.

Control goals. Stabilization. A typical goal for control of chaotic systems is stabilization of an unstable periodic solution (orbit). Let $x_*(t)$ be the T-periodic solution of the free ($u(t) = 0$) system (3) with initial condition $x_*(0) = x_{*0}$ i.e. $x_*(t+T) = x_*(t)$ for all $t \ge 0$. If the solution $x_*(t)$ is unstable, a reasonable goal is stabilization or driving solutions $x(t)$ of (3) to $x_*(t)$ in the sense of fulfillment of the limit relation

$$\lim_{t\to\infty} [x(t) - x_*(t)] = 0 \tag{15}$$

or driving the output $y(t)$ to the desired output function $y_*(t)$, i.e.

$$\lim_{t\to\infty}\left[y(t)-y_*(t)\right]=0 \qquad (16)$$

for any solution $x(t)$ of (3) with initial conditions $x(0)=x_0\in\Omega$, where Ω is a given set of initial conditions.

The problem is to find a control function in the form of either an open-loop (feedforward) control

$$u(t)=U(t,x_0) \qquad (17)$$

or in the form of state feedback

$$u(t)=U(x(t)) \qquad (18)$$

or output feedback

$$u(t)=U(y(t)) \qquad (19)$$

to ensure the goal (15) or (16).

Such a problem is nothing but a tracking problem standard for control theory. However the key feature of the control of chaotic systems is to achieve the goal by means of sufficiently small (ideally, arbitrarily small) control. Solvability of this task is not obvious since the trajectory $x_*(t)$ is unstable.

A special case of the above problem is stabilization of the unstable equilibrium x_{*0} of system (3) with $u=0$, i.e. stabilization of x_{*0}, satisfying $F(x_{*o},0)=0$. Again, this is just the standard regulation problem with an additional restriction that "small control" solutions are sought. Such a restriction makes the problem far from standard: even for a simple pendulum, nonlocal solutions of the stabilization problem with small control are nontrivial. The class of admissible control laws can be extended by introducing dynamic feedback described by differential or time-delayed models. Similar formulations hold for discrete and time-delayed systems.

Chaotization. A second class of control goals corresponds to the problems of *excitation* or *generation* of chaotic oscillations (also called *chaotization*, *chaotification* or *anticontrol*). Sometimes these problems can be reduced to the form (16), but the goal trajectory $x_*(t)$ is no longer periodic, while the initial state is equilibrium. The goal trajectory may be specified only partially. Otherwise, the goal may be to meet some formal criterion of chaos, e.g. positivity of the largest Lyapunov exponent.

Synchronization. Third important class of control goals corresponds to *synchronization* (more accurately, *controlled synchronization* as opposed to *autosynchronization* or *self-synchronization*). Generally speaking, synchronization is understood as concordance or concurrent change of the states of two or more systems or, perhaps, concurrent change of some quantities related to the systems, e.g. alignment of oscillation frequencies. If the required relation is established only asymptotically, one may speak about *asymptotic synchronization*. If synchronization does not exist in the system without control ($u=0$) the following *controlled synchronization* problem may be posed: find a control function $u(t)$ ensuring synchronization in the closed-loop system. In this case synchronization is the control goal. For example, the goal corresponding to asymptotic synchronization of the two system states x_1 and x_2 can be expressed as follows:

$$\lim_{t\to\infty}\left[x_1(t)-x_2(t)\right]=0. \qquad (20)$$

In the extended state space $x=\{x_1,x_2\}$ of the overall system, relation (20) implies convergence of the solution $x(t)$ to the diagonal set $\{x : x_1 = x_2\}$.

Asymptotic identity of the values of some quantity $G(x)$ for two systems can be formulated as

$$\lim_{t\to\infty}\left[G\left(x_1(t)\right)-G\left(x_2(t)\right)\right]=0. \qquad (21)$$

Goal functions. To solve a control problem it is often convenient to rewrite the goals (15), (16), (20) or (21) in terms of appropriate goal function $Q(x,t)$ as follows:

$$\lim_{t\to\infty}Q\left(x(t),t\right)=0. \qquad (22)$$

For example, to reduce goal (20) to the form (22) one may choose

$$Q(x)=|x_1-x_2|^2.$$

Instead of Euclidean norm other quadratic functions can also be used, e.g. for the case of the goal (15) the goal function

$$Q(x,t)=\left[x-x_*(t)\right]^{\mathrm{T}}\Gamma\left[x-x_*(t)\right],$$

where Γ is a positive definite symmetric matrix, "*T*" stands for matrix transposition can be used. The choice of the matrix Γ provides the possibility of weighting different components of the system state vector to take into account differences in their scale or importance.

In the case of chaotization problem, a goal function $G(x)$ may be introduced such that the goal is to achieve the limit inequality

$$\underline{\lim}_{t\to\infty} G\left(x(t)\right) \geq G_*. \qquad (23)$$

Typical choice of the goal function for chaotization is the largest Lyapunov exponent: $G = \lambda_1$ with $G_* > 0$. In some cases the total energy of mechanical or electrical oscillations can serve as $G(x)$.

In terms of goal functions more subtle control goals can be specified, e.g. the control goal may be to modify a chaotic attractor of the free system in the sense of changing some of its characteristics (Lyapunov exponents, entropy, fractal dimension, etc). The freedom of choice of the goal function can be utilized for design purposes.

(see *Elements of control systems, Stability concepts, Popov and circle criterion*)

4. Methods of Controlling Chaos: Continuous-time Systems

4.1. Feedforward Control by Periodic Signal

Methods of *feedforward* control (also called *nonfeedback* or *open-loop* control) change the behavior of a nonlinear system by applying a properly chosen input function $u(t)$ – external excitation. Excitation can reflect influence of some physical action, e.g. external force/field/signal, or it can be some parameter perturbation (modulation). In all cases the value $u(t)$ depends only on time and does not depend on current measurements of the system variables. Such an approach is attractive because of its simplicity: no measurements or extra sensors are needed. It is especially advantageous for ultrafast processes at the molecular or atomic level where no possibility of system variable online measurements exists.

The possibility of significant changes to system dynamics by periodic excitation has been known, perhaps, since the beginning of the 20th century. A. Stephenson discovered in 1908 that a high frequency excitation can stabilize the unstable equilibrium of a pendulum. Later theoretical results and experiments of P. Kapitsa, N.N. Bogoljubov in the 1940s-1950s triggered the development of vibrational mechanics and vibrational control. Analysis of general nonlinear systems affected by high frequency excitation is based on the Krylov-Bogoljubov averaging method. According to the averaging method stability analysis of a periodically excited system is reduced to analysis of the simplified averaged system. The method provides conditions guaranteeing approximate stabilization of the given equilibrium or the desired (goal) trajectory. A related form of averaging method deals with systems excited by stochastic disturbance (dither). Accuracy of averaging method increases if excitation contains high-frequency harmonics. For physical systems it implies high forcing amplitudes.

The possibility of transforming a periodic motion into chaotic one and vice versa by means of periodic excitation of medium level was demonstrated by V. Alexeev and A.Loskutov in 1985 for a fourth-order system describing dynamics of two interacting populations. The results were based on computer simulations. In 1990 R. Lima and M. Pettini studied Duffing-Holmes oscillator

$$\ddot{\varphi} - c\varphi + b\varphi^3 = -a\dot{\varphi} + d\cos(\omega t) \qquad (24)$$

by Melnikov method. The right-hand side of (24) was considered as a small perturbation of the unperturbed Hamiltonian system. The Melnikov function related to the rate of change of the distance between stable and unstable manifolds for small perturbations was calculated analytically and parameter values producing chaotic behavior of the system were chosen. Then additional excitation was introduced into the parameter of nonlinearity $b \to b(1+\eta\cos\Omega t)$ and a new Melnikov function was computed and studied numerically. It was shown that if Ω is close to the frequency of initial excitation ω then chaos may be destroyed. Experimental confirmation of this phenomenon was made by L. Fronzoni et al. in 1991 using a magnetoelastic device with two permanent magnets, an electromagnetic shaker and an optical sensor.

Similar approach was applied by R. Chacon in 1999 to a general model of one-degree-of-freedom nonlinear oscillator with damping excited with a biharmonic forcing. The relation between damping strength and forcing amplitudes guaranteeing either chaotic or periodic behavior of the given trajectory of the excited system was obtained. Since Melnikov method leads to intractable calculations for state dimensions greater than two, analytical results are known only for systems with one degree of freedom. For higher dimensions computer simulations are used. The general problem of finding analytic conditions for creation or suppression of chaos by feedforward periodic excitation of small or medium level still remains open.

The applications of feedforward control of chaos to control of CO_2 lasers, Josephson junctions, liquid crystal models, bistable mechanical devices, circular yttrium-ion-garnet films, Murali-Lakshmanan-Chua electronic circuit, FitzHugh-Nagumo equations describing propagation of nerve pulses in a neuronal membrane etc. were reported.

(see *Describing function method*)

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Biographical Sketch

Alexander Lvovich Fradkov born May 22, 1948; received the Diploma degree in mathematics from the Faculty of Mathematics and Mechanics of St. Petersburg State University (Dept. of Theoretical Cybernetics) in 1971; Candidate of Sciences (Ph.D.) degree in Engineering Cybernetics from St. Petersburg Mechanical Institute (now - Baltic State Technical University - BSTU) in 1975 and Doctor of Sciences degree in Control Engineering in 1986 from St. Petersburg Electrotechnical Institute.

From 1971 to 1987 he occupied different research positions and in 1987 became Professor of Computer Science with BSTU. Since 1990 he has been the Head of the Laboratory for Control of Complex Systems of the Institute for the Problems of Mechanical Engineering of Russian Academy of Sciences. He is also a part time professor with the Faculty of Mathematics and Mechanics of St. Petersburg State University (Dept. of Theoretical Cybernetics). His research interests are in fields of nonlinear and adaptive control, control of oscillatory and chaotic systems and mathematical modeling. He is also working in the borderland field between Physics and Control (Cybernetical Physics).

Dr. Fradkov is coauthor of more than 300 journal and conference papers, 9 patents, 15 books and textbooks, including:

INTRODUCTION TO CONTROL OF OSCILLATIONS AND CHAOS (with A.Yu. Pogromsky, Singapore: World Scientific, 1998), NONLINEAR AND ADAPTIVE CONTROL OF COMPLEX

SYSTEMS (with I.V. Miroshnik and V.O. Nikiforov, Dordrecht: Kluwer, 1999) SELECTED CHAPTERS OF AUTOMATIC CONTROL THEORY with MATLAB examples (with B.R. Andrievsky, St. Petersburg: Nauka, 1999, in Russian) ELEMENTS OF MATHEMATICAL MODELING IN SOFTWARE ENVIRONMENTS MATLAB 5 AND SCILAB (with B.R. Andrievsky, St. Petersburg: Nauka, 2001, in Russian). Dr. Fradkov is the Vice-President of the St. Petersburg Informatics and Control Society since 1991, Member of the Russian National Committee of Automatic Control, IEEE Senior Member. Dr. Fradkov was Co-Chairman of 1st-9th International Baltic Student Olympiades on Automatic Control in 1991-2002; NOC Chairman of the 1st and 2nd International IEEE-IUTAM Conference "Control of Oscillations and Chaos" in 1997 and 2000; NOC Chairman of the 5th IFAC Symposium on Nonlinear Control Systems (NOLCOS'01). He was an associate editor of European Journal of Control (1998-2001); a member of IEEE Control Systems Society Conference Editorial Board (1998-2002); a member of the IFAC Technical Committees on Education and Nonlinear Control.

Dr.Fradkov was awarded William Girling Watson Scholarship in Electrical Engineering (University of Sydney, 1995) and JSPS Fellowship for Research in Japan in 1998-1999. During 1991-2001 he visited and gave invited lectures in more than 60 Universities of 20 countries.